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Flutter of an Elastic Plate under Tension

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THIS problem is reconsidered and a new interpretation of existing results presented. For simplicity, two-dimensional aerodynamic flow over a one-dimensional (infinitely wide) plate is considered. However, the neglect of finite width should not alter the qualitative nature of the results or the essential conclusions that are reached.[1] The primary purpose is to clarify the effect of variations in plate bending stiffness on the flutter of a plate under a given tension, and vice versa. The particular limit as the plate bending stiffness vanishes gives a membrane whose flutter behavior has led to much discussion in the literature. In this limit, for sufficiently large supersonic Mach numbers where quasi-static Ackeret aerodynamics may be used and coupled mode flutter would occur for a plate with finite bending stiffness, the theory predicts that no flutter will occur. Spriggs et al.[2] have investigated this limit carefully and shown that for small (but nonvanishing) bending stiffness flutter will occur when

$$\lambda\epsilon^3 > (2/3)^{3/2} \tag{1}$$

where

$\lambda \equiv 2qa^3/\beta D$
$\epsilon^3 \equiv (D/a^2N_x)^{3/2}$
a ≡ plate length
D ≡ plate bending stiffness
N_x ≡ tension
q ≡ dynamic pressure of aerodynamic flow
$\beta \equiv (M^2-1)^{1/2}$
M ≡ flow Mach number

Inequality (1) holds for $\epsilon \to 0$ (as we shall see for $\epsilon \lesssim 0.025$) when the tension-induced stiffness is much larger than the bending stiffness. Inequality (1) may be rewritten as[3,4]

$$2qD^{1/2}/\beta N_x^{3/2} > (2/3)^{3/2} \tag{1a}$$

which is independent of plate length a and, moreover, shows that as D becomes smaller a larger q is required to initiate flutter. Indeed, as $D \to 0$, the q required for flutter approaches infinity. This is the essence of the "membrane paradox."[2]

It should be noted that Spriggs et al.[2] have chosen to introduce the membrane stress σ_x, which is related to N_x by $N_x = \sigma_x h$, where h is the plate thickness. They further write the bending stiffness as $D = Eh^3/12(1-\nu^2)$, where E is the modulus of elasticity and ν is Poisson's ratio. If one holds σ_x constant, then inequality (1) is independent of h, and, as $h \to 0$ [and hence the bending stiffness $D \equiv Eh^3/12(1-\nu^2) \to 0$], a finite q is obtained from inequality (1) which is required to initiate flutter. In this sense, there is no "membrane paradox." Although the foregoing is perfectly correct mathematically, the physical choice of maintaining constant σ_x rather than constant N_x seems less likely to be appropriate in most applications. One would expect a constant N_x to be imposed in most cases (independent of h), and this is the approach followed here.

Further insight into this remarkable result can be gained by considering the flutter boundary in terms of λ and ϵ for the full range of ϵ. For $\epsilon \to \infty$, it is well known from the work of Hedgepeth[5] and subsequent authors that flutter occurs when (for simply supported leading and trailing edges)

$$\lambda > 343.3 \tag{2}$$

For intermediate ϵ, the results may be presented graphically using the numerical data of Dugundji,[6] Dixon,[7] and Spriggs et al.[2] In Fig. 1, the flutter boundary is shown in terms of λ vs $1/\epsilon$. This presentation is appropriate for a plate of a given bending stiffness where the effect of tension on the flutter boundary is being considered. As can be seen, as the tension $1/\epsilon$ increases,

$$1/\epsilon \equiv (N_x a^2/D)^{1/2}$$

the dynamic pressure required for flutter to occur, $\lambda \equiv 2qa^3/\beta D$, also increases monotonically. Here, because D is held fixed and N_x varied, no membrane paradox is evident.

Now consider the same data presented in Fig. 2 in terms of $\lambda\epsilon^2 \equiv 2qa/\beta N_x$ vs $\epsilon \equiv (D/N_x a^2)^{1/2}$. Also shown are the asymptotic results given by Eqs. (1) and (2). Here the tension N_x is considered fixed and the bending stiffness D is varied. The results shown in Fig. 2 display a result at variance with one's intuition. There is a minimum in the dynamic pressure required for flutter at an associated value for bending stiffness, $\epsilon \simeq 0.1$. For values of bending stiffness larger or smaller than $\epsilon \simeq 0.1$, the dynamic pressure required for flutter is in-

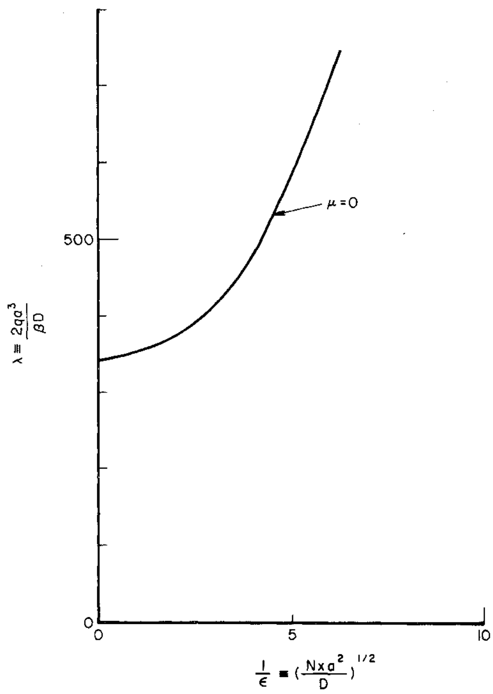

Fig. 1 Flutter boundary in terms of λ vs $1/\epsilon$.

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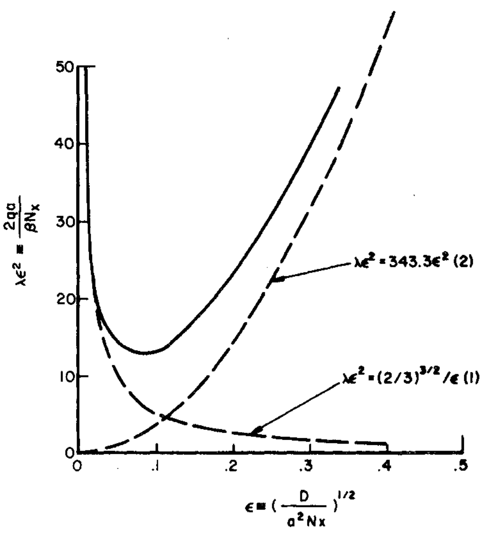

Fig. 2 Flutter boundary in terms of $\lambda\epsilon^2 \equiv 2qa/\beta N_x$ vs $\epsilon \equiv (D/N_x a^2)^{1/2}$.

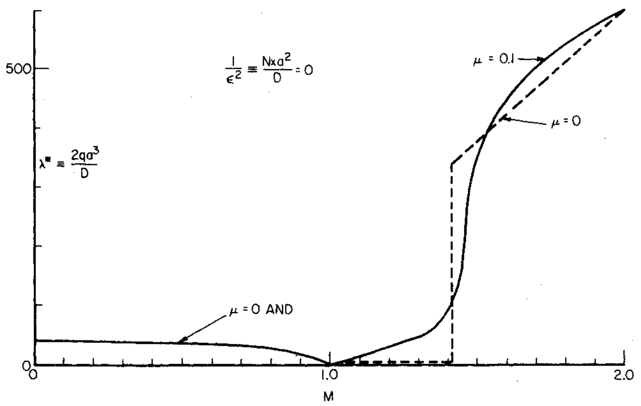

Fig. 3 Dynamic pressure at which flutter occurs vs M for zero tension.

creased. As the value of bending stiffness becomes very large or very small compared to the value that minimizes the q required for flutter, the flutter q is increased indefinitely. Figure 2 has not been displayed previously in the literature, although the numerical data from which it derives have been available for several years.

It should be noted that Spriggs et al.[2] presented the results of the present Figs. 1 and 2 in terms of ϵ vs $\lambda\epsilon^3$ (see Fig. 2 of Ref. 2). Implicit in this presentation was the result of Fig. 2 of the present paper. More explicitly, these authors commented, concerning their Fig. 2, that "it is also interesting to note that it is possible for a very thin panel to be destabilized by increasing [its] reduced [bending] modulus....This result may seem to contradict one's intuition, but perhaps is to be expected since the [membrane] paradox has always suggested that in some sense plates are less stable than membranes."

In examining Fig. 2, one is tempted to conclude that one way to insure that flutter will not occur is to place the plate under a tension load and reduce the bending stiffness until the q required to initiate flutter exceeds that available in the aerodynamic flow. However, there are several reasons why this may not be possible. In practical terms, a certain bending stiffness may be required for reasons having nothing to do with flutter. But there is a more fundamental limitation based solely on flutter considerations as follows. The results of Fig. 2 are for sufficiently larger supersonic Mach numbers. However, the critical Mach number range for most applications is in the transonic low-supersonic range, where the dynamic pressure required for flutter is smallest. At low-

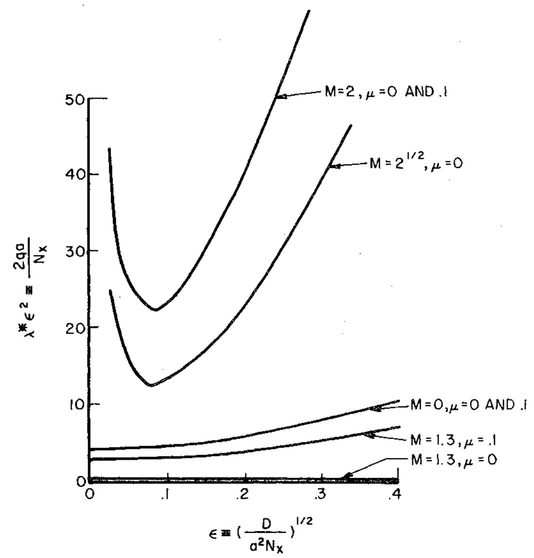

Fig. 4 Representative results for $M = 0$, 1.3, and 2.

supersonic transonic Mach numbers, the flutter condition depends upon Mach number M and dynamic pressure $\lambda^* \equiv 2qa^3/D$ separately; it also depends on a new parameter, the mass ratio $\mu \equiv \rho a/m$, as well as upon ϵ as before. Here $\rho \equiv$ flow density, and $m \equiv$ plate mass/area. In Fig. 3 (from Ref. 8) the dynamic pressure at which flutter occurs, $\lambda^* \equiv 2qa^3/D$, is shown vs M for zero tension, $1/\epsilon \equiv 0$; results for two different values of mass ratio are displayed, $\mu = 0$ and 0.1.

For small μ, $\lambda^* \sim \mu$ in the transonic/low-supersonic range. For subsonic Mach number, λ^* is independent of μ, and for high supersonic Mach number, the results depend only weakly on μ when it is small.‡ Moreover, it is known that the flutter mode for a one-dimensional, infinitely wide plate is essentially the first plate natural mode for subsonic and low supersonic Mach numbers.[8] Most important, it is known that the dynamic pressure required for flutter is proportional to the square of the first natural mode frequency[8,9]; but the latter is the natural frequency of a plate without tension times the following factor (for a simply supported plate), which includes the effect of the tension:

$$(1+N_x a^2)/D\pi^2 = 1 + (1/\epsilon^2\pi^2)$$

Using this result,§ the data of Fig. 3 may be cross-plotted to obtain results comparable to Fig. 2 for various Mach numbers. Representative results are shown in Fig. 4 for $M = 0$, 1.3, and 2. As is seen clearly from this figure, there is no membrane paradox for $M = 0$ and 1.3. At these Mach numbers, a reduction of bending stiffness (for a given tension) monotonically reduces the dynamic pressure required for flutter. Hence, for low-supersonic/transonic Mach numbers where flutter is most likely to occur, reducing bending stiffness reduces the dynamic pressure required for flutter. This is also true at subsonic Mach number, of course.

Acknowledgment

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‡Hence μ was ignored in our earlier discussion for high supersonic M.

§This simple result holds strictly only for simply supported edges where the addition of tension changes the plate natural frequency but not its mode shapes.

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Transient Ablation of Teflon Hemispheres

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Introduction

FOR high-speed entry of space vehicles into atmospheric environments, ablation is a practical method for alleviating severe aerodynamic heating. Several studies[1-3] have been undertaken on steady or quasi-steady ablation. However, ablation is a very complicated phenomenon in which a nonequilibrium chemical process is associated with an aerodynamic process that involves changes in body shape with time. Therefore, it seems realistic to consider that ablation is an unsteady phenomenon. In the design of an ablative heat-shield system, since the ultimate purpose of the heat shield is to keep the internal temperature of the space vehicle at a safe level during entry, the transient heat conduction characteristics of the ablator may be critical in the selection of the material and its thickness.

Kindler[4] studied Teflon ablation in arc-heated airflow. He measured the internal temperature and recession depth along the centerline during ablation. However, the measurements are not sufficient to explain completely the thermal behavior of the heat-shield material. This Note presents an experimental study of transient ablation of Teflon, with particular emphasis on the change in body shape, the instantaneous internal temperature distribution, and the effect of thermal expansion on ablation rate.

Experiment

The experiment was performed in a high-enthalpy hypersonic wind tunnel of the blow-down type.[3] The freestream conditions were: Mach number $M_\infty = 5.74$, stagnation temperature $T_{st} = 1020°C$, and stagnation pressure $P_{st} = 1$ atm.

Hemisphere-cylinder Teflon models were made by compressing Teflon powder containing a temperature sensor under high temperature. The diameter of the cylinder was 20 mm. The temperature sensor was a chromel-alumel thermocouple, each element being 0.1 mm in diameter. Since it was difficult to install several sensors in a model, either one thermocouple or several were buried in each model. The whole temperature distribution was obtained by superposition of the temperature histories in each experiment. Soft x-ray photography was used to check the state of the buried thermocouples and the position of their junction.

Since the ablation phenomenon is essentially transient with the change in body shape, continuous data on the change in shape are required. In this experiment, the origin of the time axis is when the valve of the wind tunnel opens. We considered two methods for measuring the ablation rate (recession velocity): 1) after the wind-tunnel valve closes, the ablation rate is measured; and 2) instantaneous shapes of an ablating model are measured by photographs taken every few seconds. The apparent ablation rate measured from the photographs is corrected by subtracting the effect of thermal expansion. In the first method, even though the valve is closed, the heat transfer to the model is not cut off instantaneously and completely. Furthermore, the model shortening measured after the test is completed, i.e., when the model cools to its initial temperature, is not exact. Some

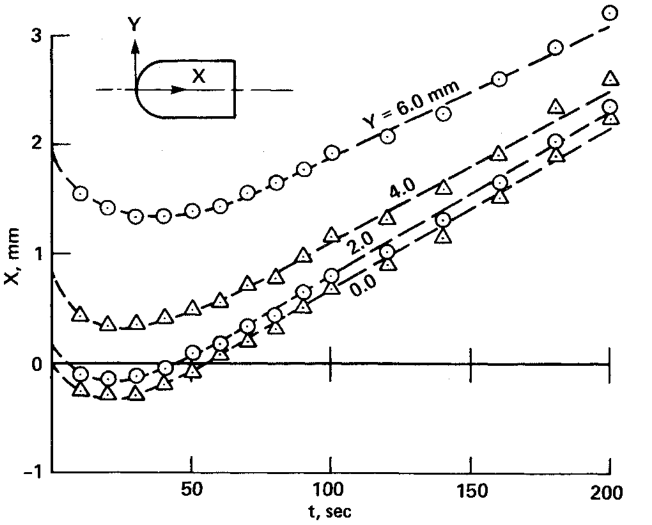

Fig. 1 Recession depth history.

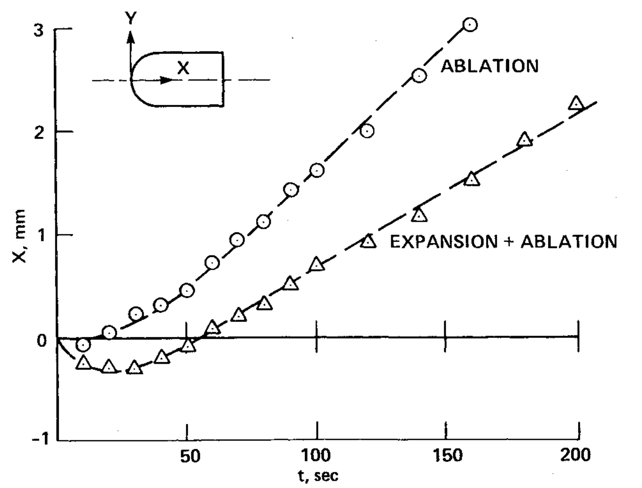

Fig. 2 Modified recession depth history at stagnation point.

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